Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. (2009)

doi:10.1111/j.1365-2966.2009.15906.x

Constraining the initial mass function of stars in the Galactic Centre

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Accepted 2009 October 20. Received 2009 September 12; in original form 2009 July 8

ABSTRACT

For half a century, evidence has been growing that the formation of stars follows a universal distribution of stellar masses. In fact, no stellar population has been found showing a systematic deviation from the canonical initial mass function (IMF) found for example for the stars in the solar neighbourhood. The only exception may be the young stellar discs in the Galactic Centre, which have been argued to exhibit a top-heavy IMF.

Here we discuss the question whether the extreme circumstances in the centre of the Milky Way may be the reason for a significant variation of the IMF. By means of stellar evolution models using different codes, we show that the observed luminosity in the central parsec is too high to be explained by a long-standing top-heavy IMF as suggested by other authors, considering the limited amount of mass inferred from stellar kinematics in this region. In contrast, continuous star formation over the Galaxy's lifetime following a canonical IMF results in a mass-to-light ratio and a total mass of stellar black holes (SBHs) consistent with the observations. Furthermore, these SBHs migrate towards the centre due to dynamical friction, turning the cusp of visible stars into a core as observed in the Galactic Centre. For the first time here we explain the luminosity and dynamical mass of the central cluster and both the presence and extent of the observed core, since the number of SBHs expected from a canonical IMF is just enough to make up for the missing luminous mass.

We conclude that observations of the Galactic Centre are well consistent with continuous star formation following the canonical IMF and do not suggest a systematic variation as a result of the region's properties such as high density, metallicity, strong tidal field etc. If the young stellar discs prove to follow a top-heavy IMF, the circumstances that led to their formation must be very rare, since these have not affected most of the central cluster.

Key words: black hole physics – stars: formation – stars: luminosity function, mass function – Galaxy: centre.

1 INTRODUCTION

In 1955, Salpeter found that the initial mass distribution of field stars in the range of $0.4 \lesssim M_\star/\mathrm{M}_\odot \lesssim 10$ is a power law with exponent 2.35. Since then, a large number of publications have investigated the initial mass function (IMF) of stars and made clear that star formation in general follows the same empirical law, the canonical IMF (Kroupa 2001, and references therein).

Due to its extreme conditions (mass density, velocity dispersion, tidal forces), the Galactic Centre provides a unique environment for testing the universality of the IMF. Star formation in the central region has thus been studied in detail; however, no agreement has been reached on the nature of the IMF in either theory or observations: Maness et al. (2007) find a best fit of observations in the central parsec of our Galaxy to a model of constant star formation with a top-heavy IMF whereas Buchholz, Schödel & Eckart (2009) show that the old stellar cluster in the Galactic Centre very well resembles the bulge population. Observations of the young, massive Arches cluster in the central region of the Milky Way have long been interpreted as a prime example for top-heavy star formation (e.g. Figer et al. 1999; Stolte et al. 2002; Kim et al. 2006; Klessen, Spaans & Jappsen 2007). However, Espinoza, Selman & Melnick (2009) have shown that a canonical IMF cannot be excluded for this cluster. Paumard et al. (2006) suggested a flat IMF for the young OB stars observed in discs in the central parsec from the analysis of the *K*-band luminosity function (LF). Based on more recent spectroscopic observations, Bartko et al. (2009b) find strong evidence for this to be true. Bonnell & Rice (2008) found from smoothed particle hydrodynamics (SPH) simulations that the IMF of stars forming in fragmenting accretion discs strongly depends on the parameters of

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the underlying gas infall scenario. Unfortunately, theoretical IMF predictions have failed in the past to correctly describe the observations near the Galactic Centre (Kroupa 2008a).

In this paper, we combine observational data with models of stellar evolution and dynamics to constrain the stellar mass function and star formation history in the Galactic Centre. It is organized as follows. In Section 2, we analyse the properties of models of the Galactic Centre assuming different star formation histories and compare them to the observations. Section 3 describes the mass profile of the central parsec and the effect of mass segregation. We discuss the IMF of the young stellar discs around Sgr A* and appropriate formation scenarios in Section 4 and summarize in Section 5.

2 STELLAR EVOLUTION MODELS OF THE GALACTIC CENTRE

Observations of the central parsec of the Milky Way show that this region is dominated by a dense population of old stars with a total mass of $\sim 1.5 \times 10^6\,\mathrm{M}_\odot$ (Genzel et al. 2003; Schödel et al. 2007; Buchholz, Schödel & Eckart 2009; Schödel, Merritt & Eckart 2009). Within the uncertainties of a factor of 2, Schödel et al. (2009) find that the extended mass inferred from kinematics can be explained well by the visible stars. On the other hand, if star formation in the Galactic Centre occurs following a top-heavy IMF as suggested by Maness et al. (2007), one would expect a large number (and thus significant mass) of dark remnants.

To test which mass functions are consistent with the observations, we used the stellar evolution package SSE (Hurley, Pols & Tout 2000) to calculate population synthesis models after 13 Gyr of star formation. Assuming (broken) power-law IMFs of the form $\xi(m) \propto m^{-\alpha}$, where $\xi(m)\,\mathrm{d}m$ is the number of stars in the mass interval m to $m + \mathrm{d}m$, we used the following models.

(a) The canonical IMF according to Kroupa (2001), $\xi(m) \propto m^{-\alpha_i}$, with $\alpha_0 = 0.3 (0.01 \le m/\mathrm{M}_\odot < 0.08), \alpha_1 = 1.3 (0.08 \le m/\mathrm{M}_\odot < 0.5)$ and $\alpha_2 = 2.3 (0.5 \le m/\mathrm{M}_\odot \le 120)$.

(b) A flat IMF with $\alpha = 1.35 (1 \le m/\mathrm{M}_\odot \le 120)$ as suggested by Paumard et al. (2006) for the young stellar discs in the Galactic Centre.

(c) The same IMF, but extended to $0.01 \le m/\mathrm{M}_\odot \le 120$.

(d) $\alpha = 0.85$ as suggested by Maness et al. (2007), again for $0.01 \le m/\mathrm{M}_\odot \le 120$.

Starting with solar metallicity ($Z = 0.02$), we calculated evolutionary tracks for stars with masses of $0.01 \le m/\mathrm{M}_\odot \le 120$ in steps of $0.01\,\mathrm{M}_\odot$. We averaged the results over time, weighted with different star formation histories of the form $\mathrm{SFR}(t) \propto \mathrm{e}^{-t/t_0}$: we used constant ($t_0 = \infty$), exponentially declining ($t_0 = 3\,\mathrm{Gyr}$, $t_0 = 1\,\mathrm{Gyr}$, $t_0 = 300\,\mathrm{Myr}$) and exponentially increasing star formation rates (SFR; $t_0 = -3\,\mathrm{Gyr}$). Weighing the outcome with any of the above IMFs gives the total mass fraction of neutron stars (NSs) and stellar black holes (SBHs), as well as the total numbers of NSs and SBHs in the central parsec (assuming an enclosed mass of $1.5 \times 10^6\,\mathrm{M}_\odot$; Schödel et al. 2009). To estimate the total number of bright stars (mag $_{Ks} \lesssim 17.5$) and the K-band mass-to-light ratio M/L_{Ks} of the unresolved stars and stellar remnants, we generated a sequence of Padova isochrones (Bertelli et al. 1994; Marigo et al. 2008), assuming an average extinction of 3.3 mag for stars in the Galactic Centre (Schödel et al. 2009). We used the magnitudes calculated for the Two-Micron All-Sky Survey (2MASS) *Ks* filter (Cohen, Wheaton & Megeath 2003), whose transmission curve closely resembles that of the *Ks*-band filter of the NAOS-CONICA instrument at the European Southern Observatory (ESO) Very Large Telescope (VLT), which was used for the observations of the Galactic Centre discussed here. We further assumed that the mass of a stellar remnant depends on the initial mass as

$$m_{\mathrm{rem}} = \begin{cases} 0.109\, m_{\mathrm{init}} + 0.394\,\mathrm{M}_\odot, & 0.8 < m_{\mathrm{init}}/\mathrm{M}_\odot < 8, \\ 1.35\,\mathrm{M}_\odot & 8 \le m_{\mathrm{init}}/\mathrm{M}_\odot < 25, \\ 0.1\, m_{\mathrm{init}}, & 25 \le m_{\mathrm{init}}/\mathrm{M}_\odot \end{cases} \qquad (1)$$

(Thorsett & Chakrabarty 1999; Baumgardt & Mieske 2008; Kalirai et al. 2008; Dabringhausen, Kroupa & Baumgardt 2009). The uncertainties of these assumptions are not easily quantified, since the underlying theory and observations are not robust, but using the masses, radii and effective temperatures from our SSE models and assuming a blackbody spectrum lead to comparable results. We are thus confident that our results are correct within a factor much less than 2.

Table 1 lists the respective values for all four models. While most stars survive the age of the Galaxy as main-sequence stars in the canonical model (a), an old cluster based on a top-heavy IMF is mass-dominated by stellar black holes. Schödel et al. (2009) find that the unresolved stellar population makes up >98 per cent of the mass in the central parsec and find its mass-to-light ratio to be $M/L_{Ks} = 1.4^{+1.4}_{-0.7}\,\mathrm{M}_\odot/\mathrm{L}_{\odot,Ks}$. Fig. 1 plots the ratio of total mass to diffuse light in our models as a function of the star formation history parameter t_0. We find that the observations are consistent with the canonical IMF (a), with a tendency towards constant or increasing star formation. The $\alpha = 1.35$ models (b, c) require increasing star formation, and an IMF as flat as $\alpha = 0.85$ (d) is not consistent with the observed old population at all.

We can also compare the results of our analysis as given in Table 1 to stellar number counts in the central parsec. Assuming a canonical IMF, we expect $\sim 2.5 \times 10^4$ SBHs and NSs for every $1.5 \times 10^6\,\mathrm{M}_\odot$ in stars and stellar remnants, the latter being the estimated enclosed mass within 1 pc from Sgr A*. Due to dynamical friction, SBHs may migrate to the central parsec from as far as 6 pc from the centre within 10 Gyr (see Section 3 for details), thus increasing the number of SBHs in the central parsec by up to one order of magnitude. On the other hand, SBHs may spiral into the supermassive black hole (SMBH) as discussed above. In total, the expected number of SBHs agrees best with Muno et al. (2005) suggesting a number of $\sim 10^4$ SBHs and NSs in this region from X-ray observations, while models based on a top-heavy IMF suggest numbers well above 10^5.

A more verifiable quantity is the number of bright stars: Schödel et al. (2009) find ~ 6000 stars with a *Ks*-band magnitude of $\lesssim 17.5$ within a projected distance of 1 pc from Sgr A*. Assuming a spherical distribution with a density profile $\rho \propto r^{-1.75}$ (Schödel et al. 2007), ~ 3000 of these stars are in the innermost parsec. This value is consistent with our model (a) of a canonical IMF, assuming a constant or declining SFR, or with models (b) and (c) assuming constant or slightly decreasing star formation ($t_0 > 3\,\mathrm{Gyr}$). An even flatter IMF (d) requires a SFR increasing with time to explain the observed number of luminous stars.

Buchholz et al. (2009) find that the K-band LF of late-type stars in the central parsec closely resembles that of the bulge population. Here we use the luminosities calculated from our stellar evolution models to compare the LFs of different IMFs. Fig. 2 shows the K-band LFs of our models as a function of star formation history. It is seen that the shape of the LF does not depend significantly on the IMF. In particular, all curves can be approximated by a power law with a slope $\beta \approx 0.3$, and all exhibit the horizontal branch/red clump peak. This peak is offset to the observations of Buchholz

Table 1. Dependence of the composition of the Galactic Centre after 13 Gyr of star formation on the IMF and star formation history.

Model	IMF	Mass range	t_0	$M_{\rm NS}/M_{\rm tot}$ (per cent)	$M_{\rm BH}/M_{\rm tot}$ (per cent)	$N_{\rm NS}$	$N_{\rm BH}$	$N_{{\rm mag}_{Ks}<17.5}$	$M/L_{\rm diffuse}$
(a)	Canonical	$0.01 \le m/{\rm M}_\odot \le 120$	−3 Gyr	1.74	3.84	16 861	6294	11 442	1.86
			∞	1.86	4.09	18 064	6713	7166	2.65
			3 Gyr	1.94	4.26	18 832	6989	4371	3.59
			1 Gyr	1.97	4.31	19 060	7074	3736	3.95
			300 Myr	1.97	4.32	19 121	7096	3630	4.03
(b)	$\alpha = 1.35$	$1 \le m/{\rm M}_\odot \le 120$	−3 Gyr	8.73	67.80	80 853	95 223	18 625	2.15
			∞	9.31	71.98	86 192	101 100	8740	4.76
			3 Gyr	9.60	74.14	88 879	104 133	3152	17.91
			1 Gyr	9.66	74.60	89 438	104 781	1497	74.79
			300 Myr	9.67	74.67	89 519	104 877	226	942.30
(c)	$\alpha = 1.35$	$0.01 \le m/{\rm M}_\odot \le 120$	−3 Gyr	7.27	56.43	67 295	79 255	14 107	2.44
			∞	7.67	59.31	71 025	83 309	6538	4.45
			3 Gyr	7.87	60.78	72 868	85 374	2632	8.04
			1 Gyr	7.91	61.11	73 262	85 830	1966	9.61
			300 Myr	7.92	61.18	73 348	85 931	1878	9.95
(d)	$\alpha = 0.85$	$0.01 \le m/{\rm M}_\odot \le 120$	−3 Gyr	5.38	85.51	48 567	111 458	6236	7.45
			∞	5.51	87.20	49 722	113 673	2350	16.35
			3 Gyr	5.56	87.89	50 167	114 567	687	37.29
			1 Gyr	5.57	88.00	50 234	114 712	458	48.00
			300 Myr	5.57	88.02	50 245	114 738	433	50.26

Note. The columns give the following in order: model name, underlying IMF (slope), range of stellar masses used; star formation history parameter, mass fraction in NSs, mass fraction in black holes, number of NSs and stellar black holes per $1.5 \times 10^6\,{\rm M}_\odot$ (i.e. within 1 pc from Sgr A*), number of bright stars (mag $_{Ks} \lesssim 17.5$), mass-to-light ratio of unresolved population. While most stars survive the age of the Galaxy in the canonical model (a), an old cluster based on a top-heavy IMF is mass-dominated by stellar black holes. In contrast to the number of remnants, the mass-to-light ratio as well as the number of bright stars strongly depend on the star formation history.

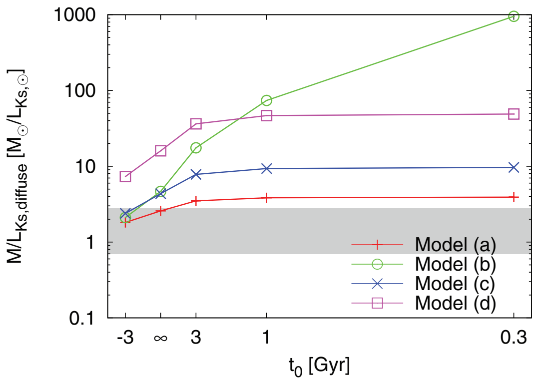

Figure 1. Ratio of total mass to diffuse light in our models as a function of the star formation history parameter t_0. The shaded area marks the 1σ range derived by Schödel et al. (2009) from observations. While observations are best explained with an increasing SFR following the canonical (a) or a moderately top-heavy (b, c; $\alpha = 1.35$) IMF, observations cannot be reproduced assuming a flatter IMF (d; $\alpha = 0.85$).

et al. (2009) by 1 mag, which may be due to our models of stellar evolution and the assumed constant extinction. We thus presume an uncertainty of a factor of 2 in the luminosity estimates discussed above, which does not affect our qualitative results and reasoning.

Altogether, observations of the old stars, diffuse light and stellar dynamics in the Galactic Centre are best explained with a canonical IMF at constant or decreasing star formation, but may also be explained with a somewhat flatter IMF and an almost constant SFR. An IMF slope flatter than $\alpha = 1$ can be safely ruled out.

We have to stress that here (and in the next section) we only calculate self-contained models, i.e. we assume that the stars now present in the Galactic Centre also formed in this region. Stars may also be brought to the vicinity of the SMBH by capture of individual stars or clusters, and some unknown fraction of the stars observed in the centre of the Milky Way may have formed in a much different environment. To this extent, we do not exactly discuss star formation in the Galactic Centre, but the formation of stars now observed in the central region, thus reaching an insight into the composition of stellar systems surrounding SMBHs.

3 MASS PROFILE IN THE GALACTIC CENTRE

Theoretical arguments and N-body simulations show that a stellar system around an SMBH evolves into a cusp with a $\gamma = 1.75$ power-law density distribution (Bahcall & Wolf 1976; Amaro-Seoane, Freitag & Spurzem 2004; Baumgardt, Makino & Ebisuzaki 2004a,b; Preto, Merritt & Spurzem 2004). Indeed, observations show that the central parsec of the Milky Way exhibits a corresponding profile (e.g. Genzel et al. 2003; Schödel et al. 2007, 2009). However, observations also reveal a deficit of stars within a few 0.1 pc from the SMBH (Genzel et al. 1996, 2003; Figer et al. 2003; Schödel et al. 2007, 2009). For example, Schödel et al. (2007) estimated the density profile of the stellar cusp in the Galactic Centre from the observed luminosity profile and kinematics as

$$\rho(r) = (2.8 \pm 1.3) \times 10^6\,{\rm M}_\odot{\rm pc}^{-3} \left(\frac{r}{0.22\,{\rm pc}}\right)^{-\gamma}, \qquad (2)$$

where r is the distance from the SMBH, $\gamma = 1.2$ inside 0.22 pc and $\gamma = 1.75$ outside 0.22 pc. Buchholz et al. (2009) find that the profile may be flatter or even slightly inverted in the central 0.2 pc, turning the cusp into a core.

The observed core in the old population around Sgr A* may be explained by stellar collisions destroying the envelopes of giants

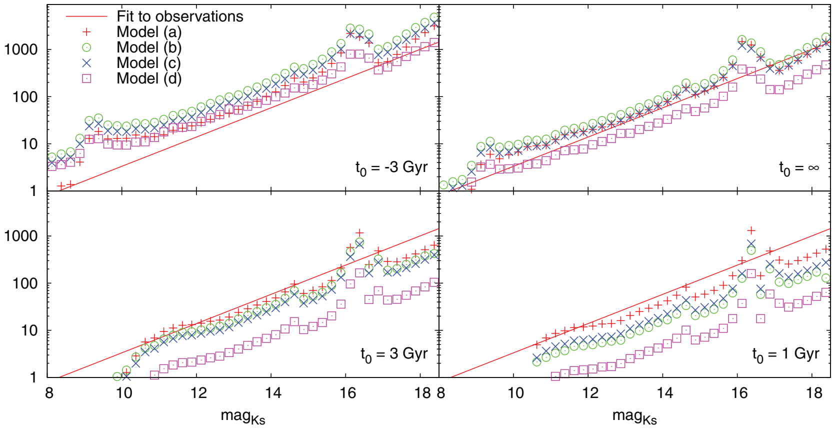

Figure 2. LF of bright stars in the Galactic Centre, assuming a K-band extinction of 3.3 mag. The panels compare the LF observed by Buchholz et al. (2009), represented by their fit as a straight line (and corrected for projection effects), to the results of our models of different IMFs following exponentially increasing (upper left), constant (upper right) and exponentially decreasing star formation (lower panels). The slope derived from the observations fits all our models quite well: it does not depend strongly on star formation history or IMF. However, as is also seen from Table 1, the normalization does depend on the SFR, especially so for the top-heavy models (b,d).

(Genzel et al. 1996; Davies et al. 1998; Alexander 1999; Bailey & Davies 1999; Dale et al. 2009) or by an inspiralling intermediate-mass black hole depleting the central region (Baumgardt, Gualandris & Portegies Zwart 2006; Merritt & Szell 2006; Löckmann & Baumgardt 2008).

Here we suggest that the central 0.2 pc around Sgr A* is not mass-depleted, but dominated by a cusp of SBHs having displaced the less massive visible stars: due to dynamical friction, the most massive stellar remnants sink into the depth of the SMBH's potential well. Thus, the SMBH is expected to be surrounded by a large number of black holes in its immediate vicinity (Baumgardt, Makino & Ebisuzaki 2004b; Freitag, Amaro-Seoane & Kalogera 2006). Freitag et al. (2006) have shown that the SBHs in the Galactic Centre establish a cusp profile $\rho(r) \propto r^{-1.75}$, while the density profile of the less massive stars becomes consistent with the observed luminosity profile. They do not find a clear density cut-off around 0.2 pc as in the observations, since they had to use an unrealistically large number of SBHs due to computational limitations of the methods used. Since it is as yet not possible to perform a full calculation including a realistic number of stars and SBHs, we here follow a theoretical approach.

For simplicity, we assume that the Galactic Centre formed evolved stars and SBHs 10 Gyr ago without any further star formation. The frictional drag on an inspiralling SBH can be estimated as

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{4\pi \ln\Lambda\, G^2 \rho_\star(r,t) M_{\mathrm{BH}}}{v^3}\left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}}\mathrm{e}^{-X^2}\right]\boldsymbol{v} \qquad (3)$$

(Binney & Tremaine 1987, equation 7-18), where $\boldsymbol{v}$ is the velocity of the SBH, G is the gravitational constant, M_{BH} is the SBH mass, $\rho_\star(r, t)$ is the density of the stellar background, $\ln\Lambda$ is the Coulomb logarithm and $X = v/(\sqrt{2}\sigma)$ is the ratio of the SBH velocity to the (1D) stellar velocity dispersion σ. For a stellar density profile $\rho_\star(r) \propto r^{-\gamma}$ with $1.2 \le \gamma \le 1.75$, one gets $X \approx 1.2$. Assuming that the overall density profile does not change with time and can be described by $\rho(r) \equiv \rho_\star(r, t) + \rho_{\mathrm{BH}}(r, t) = \rho_0 r^{-\gamma}$ yields

$$v_{\mathrm{c}}(r) = \sqrt{G\left(M_{\mathrm{SMBH}} + \frac{4\pi\rho_0}{3-\gamma} r^{3-\gamma}\right) r^{-1}} \qquad (4)$$

as the circular velocity at distance r of the SMBH of mass M_{SMBH}. Inserting into

$$-r\left|\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\right| = -\frac{Fr}{M_{\mathrm{BH}}} = \frac{\mathrm{d}L}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\sqrt{r v_{\mathrm{c}}}, \qquad (5)$$

where F and L are the frictional force and orbital angular momentum, respectively, leads to the SBH inspiral speed

$$\dot{r} = \frac{-8\pi r^{5/2} \ln\Lambda \sqrt{G}\rho_\star(r,t) M_{\mathrm{BH}} \left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}}\mathrm{e}^{-X^2}\right]}{\left(M_{\mathrm{SMBH}} + 4\pi\rho_0 \frac{4-\gamma}{3-\gamma} r^{3-\gamma}\right)\sqrt{M_{\mathrm{SMBH}} + \frac{4\pi\rho_0}{3-\gamma} r^{3-\gamma}}}, \qquad (6)$$

assuming a circular orbit of the SBH. Note that $\rho_\star(r, t)$ decreases with time due to the inspiralling SBHs increasing $\rho_{\mathrm{BH}}(r, t)$.

To integrate this equation numerically, we assume that the cluster was initially not mass-segregated, $\rho_{\mathrm{BH}}(r, 0) = 0.04\rho(r)$, where $M_{\mathrm{BH}}/M_{\mathrm{tot}} \approx 0.04$ was taken from Table 1 for a canonical IMF, and $\rho(r) = 2.8 \times 10^6\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}(r/0.22\,\mathrm{pc})^{-\gamma}$ is a Bahcall & Wolf (1976) profile with $\gamma = 1.75$. We further assume an SMBH mass of $M_{\mathrm{SMBH}} = 4 \times 10^6\,\mathrm{M}_\odot$ and SBHs of mass $M_{\mathrm{BH}} = 10\,\mathrm{M}_\odot$. Fig. 3 shows the enclosed mass in SBHs as a function of central distance, $M_{\mathrm{SBH}(<r)}$, for different times t in steps of 1 Gyr, starting with $M_{\mathrm{SBH}(<r)} \propto r^{3-\gamma}$ at $t = 0$. It is seen that the cusp is saturated by SBHs in the innermost 0.5 pc within 10 Gyr. Clearly, this is not the final answer: due to random deflections of stars, we can expect a number of stars in the innermost region. Furthermore, contrary to our above assumptions, both stars and SBHs move on eccentric orbits, preventing a strict segregation.

In fact, Freitag et al. (2006) find that the inspiralling SBHs build a Bahcall & Wolf (1976) profile, while the density profile of the stars is flatter, compatible with the slope observed in the central 0.2 pc. In particular, they find that the central part will be mass-dominated by the SBHs, suggesting that the above estimates are at least qualitatively correct.

Altogether, the above discussion suggests that

(i) the innermost region is mass-dominated by SBHs within $a \sim 0.5$ pc, the semimajor axis a being distributed as $a^{2-1.75}$;

(ii) in this region, the stars are distributed roughly as $a^{2-1.2}$;

(iii) outside $a \sim 0.5$ pc, the cluster is strongly dominated by visible stars, $\rho_\star \gtrsim 0.94\rho$ assuming a canonical IMF.

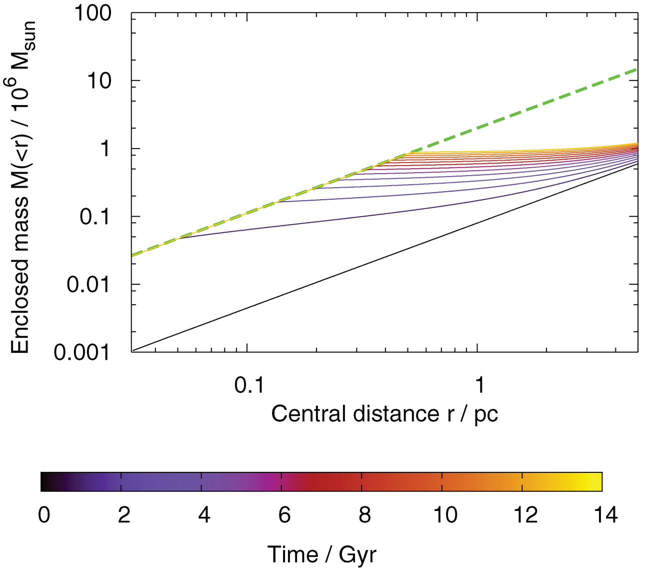

Figure 3. Mass segregation in the Galactic Centre. The solid lines show the enclosed mass in SBHs as a function of central distance in steps of 1 Gyr from the black bottom curve ($t = 0$) to the bright top curve ($t = 13$ Gyr). The dashed line depicts the total enclosed mass in SBHs and stars, which is assumed to be constant. It can be seen that the central 0.5 pc is saturated with SBHs over the Galaxy's lifetime.

To compare these results to the density profile derived from observations, we need to convert the distributions of semimajor axes into a distance distribution. For this, we assume a thermal distribution (i.e. a uniform distribution in e^2, where the eccentricity e is distributed as $f_e(e) = 2e$). A cusp with 3D 'density' of semimajor axes $\rho_a(a)$ has a 1D distribution $\phi_a(a) = 4\pi a^2 \rho_a(a)$. The central distance of a particle on a Kepler orbit with semimajor axis a and eccentricity e passes any value r (with $1 - e < r/a < 1 + e$) twice during a full orbital period T; hence, its distribution function is

$$f_r(r) = \frac{2}{T}\left|\frac{\mathrm{d}r}{\mathrm{d}t}\right|^{-1} = \frac{r}{\pi a\sqrt{a^2e^2 - (a-r)^2}}. \qquad (7)$$

A particle on an orbit of eccentricity e and semimajor axis a assumes central distances r between pericentre and apocentre distance, $a(1 - e) \le r \le a(1 + e)$. Depending on the orbital eccentricity, a particle at distance r can thus have a semimajor axis between $r/2$ and infinity. For a given semimajor axis a, the eccentricity has to be larger than $|1 - r/a|$ to be consistent with the central distance r. Altogether, the distribution function of central distances is

$$\phi_r(r) = \int_{a=r/2}^{\infty}\int_{e=|1-r/a|}^{1} f_e(e)\phi_a(a) f_r(r)\,\mathrm{d}e\,\mathrm{d}a \qquad (8)$$

$$= \int_{a=r/2}^{\infty}\int_{e=|1-r/a|}^{1} \frac{8a\,r\,e\,\rho_a(a)}{\sqrt{a^2e^2 - (a-r)^2}}\,\mathrm{d}e\,\mathrm{d}a, \qquad (9)$$

and with $\phi_r(r) = 4\,\pi r^2 \rho(r)$, the 3D density profile is

$$\rho(r) = \int_{a=r/2}^{\infty}\int_{e=|1-r/a|}^{1} \frac{2a\,e\,\rho_a(a)}{\pi r\sqrt{a^2e^2 - (a-r)^2}}\,\mathrm{d}e\,\mathrm{d}a. \qquad (10)$$

Fig. 4 shows a semimajor axis distribution $\rho_{\star,a}(a) = 2.8 \times 10^6\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}\,(0.5/0.22)^{-\gamma} \times 0.96$ of stars in the Galactic Centre as well as the resulting 3D density profile $\rho_\star(r)$. It can be seen that the density profile derived in our theory resembles the observations very well. The exact value of the break radius cannot be determined within the uncertainties of the density profile derived from the observations by Schödel et al. (2007).

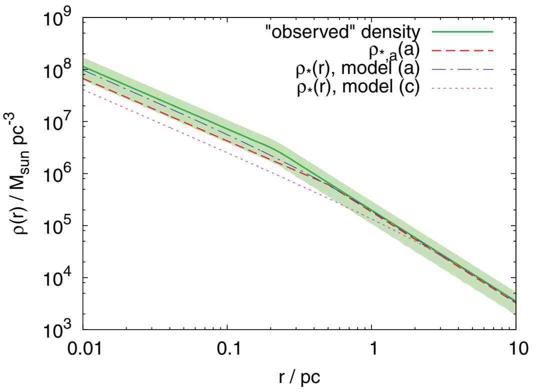

Figure 4. Density and semimajor axis distributions of stars in the Galactic Centre. The dashed line shows a semimajor axis distribution $\rho_{\star,a}(a)$ with a break radius of 0.5 pc, as predicted by theory for a canonical IMF. Assuming a thermal eccentricity distribution, the dash–dotted line shows the corresponding density profile $\rho_\star(r)$, which resembles the profile derived from observations by Schödel et al. (2007) very well. Both distributions are consistent with the observed profile within the 1σ uncertainty shown as a shaded area. In contrast, the expected density profile for a top-heavy IMF (model c in Table 1; dotted line) is not consistent with the observations, producing a break radius at 2.5 pc already after 8 Gyr of mass segregation.

On the other hand, models with a black hole mass ratio $M_{\mathrm{BH}}/M_{\mathrm{tot}} \gtrsim 0.6$, as would result from the evolution of a population of stars following a top-heavy IMF (cf. Table 1), lead to a break radius of 2.5 pc and higher already after 8 Gyr of mass segregation. Thus, the observed break radius is further evidence for a canonical IMF and a moderate black hole fraction in the Galactic Centre.

4 IMF OF THE YOUNG STELLAR DISCS

Observations of the Galactic Centre revealed one or two discs of $\approx$6 Myr old stars orbiting the central SMBH at a distance of $\sim$0.1 pc (Levin & Beloborodov 2003; Paumard et al. 2006; Lu et al. 2006, 2009; Bartko et al. 2009a; see discussion in Löckmann & Baumgardt 2009). Two main scenarios have been proposed for the formation of such massive ($M_{\mathrm{disc}} \sim 10^4\,\mathrm{M}_{\odot}$) discs close to Sgr A* as follows. Gerhard (2001) suggested that a disc of stars may have formed by tidal disruption of an infalling cluster of young stars, which would require a large mass or a central intermediate-mass black hole to survive the strong tidal forces from the SMBH (McMillan & Portegies Zwart 2003). However, Levin & Beloborodov (2003) showed that this cannot explain the small distances of the stars from Sgr A* and proposed in situ formation by fragmentation of a massive accretion disc as an alternative scenario (see also Nayakshin & Cuadra 2005). Mapelli et al. (2008) and Bonnell & Rice (2008) have shown the infall of a giant molecular cloud (GMC) towards the Galactic Centre to be effective in creating a disc of stars with distances from Sgr A* consistent with the observed young massive stars.

Paumard et al. (2006) derived a flat IMF for the observed disc stars from their K-band LF; however, this only refers to a mass interval of $20 \lesssim M_\star/\mathrm{M}_{\odot} \lesssim 30$. From the amount of mutual warping of the observed stellar discs, Nayakshin et al. (2006) derived upper mass limits to the masses of the discs considering their apparent flatness as measured by Paumard et al. (2006) after a few Myr of interaction.

However, in Löckmann, Baumgardt & Kroupa (2009) we show that a canonical IMF cannot be excluded from disc dynamics: Nayakshin et al. (2006) used stellar discs only out to 0.2 pc from the SMBH, while more than one-third of the stars listed by Paumard et al. (2006) are further away from Sgr A*. In addition, they did not distinguish between stars in the outer parts of the discs which almost retain their orbital planes and stars close to the centre which strongly precess.

Nayakshin & Sunyaev (2005) argued that the X-ray luminosity of the Sgr A* field is too low to account for the number of young $\lesssim$3 M$_\odot$ stars expected from a canonical IMF, considering the large number of O stars observed in the discs, which may be explained by a higher low-mass cut-off near 1 M$_\odot$. On the other hand, the existence of the S stars within 0.01 pc from Sgr A* is in favour of a canonical IMF (and thus a large number of B-type stars) for the stellar discs, if they were formed from these discs as suggested by Löckmann, Baumgardt & Kroupa (2008). For a more detailed discussion of the IMF of the young stellar discs, see Löckmann et al. (2009).

Assuming an infalling cluster as the origin for the stellar discs, one may expect a top-heavy mass function if the cluster was mass segregated and then tidally stripped. On the other hand, a top-heavy IMF of stars formed in a fragmenting disc would require an unusual mode of star formation. As we have discussed in the previous sections, no convincing reason or evidence for a flat IMF in the Galactic Centre in general has been found.

Using SPH simulations of star formation in fragmenting gas accretion discs, Bonnell & Rice (2008) find that the IMF of disc stars can be bimodal (and thus top-heavy) if the infalling gas cloud is massive enough ($\gtrsim 10^5$ M$_\odot$) and the impact parameter of the SMBH encounter is as small as ~0.1 pc. This way, an extreme configuration of cloud masses and distances may lead to a significant variation of the IMF.

Only recently, a systematic search of OB stars in the central parsec revealed a significant deficit of B-type stars in the regime of the young discs, suggesting a strongly top-heavy IMF for these discs (Bartko et al. 2009b). Until then, there was no strong evidence for a top-heavy IMF in the young discs. However, it cannot be assumed that the majority of stars in the Galactic Centre were formed the same way as the young disc stars, whose existence may be an indication of recently enhanced star formation processes: if the Galactic bar is young, as an increased fraction of barred galaxies for lower redshifts suggests (Sheth et al. 2008), bar-induced gas inflow may explain such an enhancement by an increasing supply of high-mass GMCs towards the central region (Sellwood & Wilkinson 1993 and references therein). Hence, despite the significant lack of B stars in the range of 0.03–0.5 pc from the SMBH, suggesting that the young discs indeed formed following a flat IMF, this does not imply that star formation in the Galactic Centre is or was in general top-heavy. Instead, the mass function of disc stars may reveal details of the formation scenario, as the results of Bonnell & Rice (2008) suggest. The majority of old stars in the region may thus have formed under different conditions (e.g. mass and impact parameter of the infalling clouds) in the scenario of a fragmenting disc, or formed further away and then migrated to the centre (as in the infalling cluster scenario, see above), or formed by any other process following the canonical IMF.

5 DISCUSSION

Various attempts have been made to study star formation in the Galactic Centre, but so far neither theory or simulations nor observations led to an agreement on the (initial) distribution of stellar masses. Here we have shown that theory and observations are consistent with star formation generally following a canonical IMF (Kroupa 2001) in the Galactic Centre, just as anywhere else in the Universe. Our main results can be summarized as follows.

(1) The mass-to-light ratio of the central parsec of the Milky Way is consistent with a constant or exponentially decreasing SFR following a canonical IMF. Models of constant star formation following an IMF with $\alpha = 1.35$ are consistent with the observed luminosities but create $\sim 10^5$ SBHs in the central parsec, 10 times more than expected by other authors. Mass functions flatter than $\alpha \approx 1$ can be safely ruled out, since they cannot explain the observed number of bright stars and the diffuse light.

(2) The core observed in the luminosity distribution with a radius of $r_{\rm break} \approx 0.2$ pc does not imply a core in the mass profile, but can be well explained by mass segregation as suggested by Freitag et al. (2006), where dark remnants mass-dominate this region. Again, the observations are best explained by a canonical IMF and are not compatible with star formation following a top-heavy IMF with $\alpha \leq 1.35$, as this would create a core radius one order of magnitude larger.

(3) Recent observations revealing a deficit of B-type stars in the young stellar discs suggest a top-heavy IMF for this population, which may be explained by tidal stripping of an infalling mass-segregated cluster, or unusual modes of star formation in a fragmenting accretion disc. However, these results do not allow conclusions on star formation in the Galactic Centre in general, for which we have no reason to assume it to be non-canonical.

While other authors generally predicted a flat IMF (e.g. Klessen et al. 2007) or a higher low-mass cut-off (e.g. Morris 1993; Levin & Beloborodov 2003; Larson 2006) from state-of-the-art theoretical star formation models of the Galactic Centre, we find that observations suggest that star formation follows a canonical IMF even under the extreme circumstances present in the central cluster. This universality of the IMF poses a major challenge to our understanding of star formation processes (see also Kroupa 2001, 2008b).

Our results rely on the assumption that the stars observed within 1 pc from the Galactic Centre also formed there, suggesting star formation with a canonical IMF even under such exotic conditions. It is possible that some of the stars were brought in by massive star clusters which spiralled towards the SMBH through dynamical friction (Portegies Zwart et al. 2006; Fujii et al. 2008). However, this scenario is unlikely because a star cluster is stripped on its way towards the centre and loses mostly low-mass stars, since it would be in a mass-segregated state soon after its formation. Therefore, the most likely scenario is a central cluster that formed over a Hubble time with a canonical IMF, where the very young stellar population of the stellar discs observed to have a very top-heavy IMF (Bartko et al. 2009b) constitutes a rare star formation event not typical for the bulk stellar population in the central cluster.

ACKNOWLEDGMENTS

This work was supported by the German Research Foundation (DFG) through the priority programme 1177 ‘Witnesses of Cosmic History: Formation and Evolution of Black Holes, Galaxies and Their Environment’.

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