Astro-286 - Week 4

1. **Gravitational focusing - runaway growth:**

   (a) Derived the mass growth rate of embryos in the case of $v_{esc} >> v_0$.

   (b) How does the mass doubling time in this regime depends on the mass?

2. **Hill Sphere** (10pt)

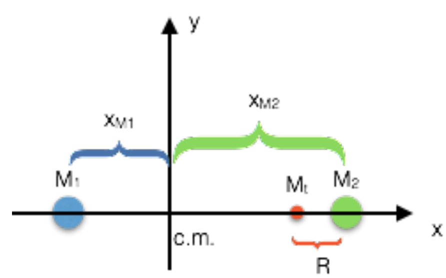
   (a) (7pt) Assume a system with two masses M_1 and M_2 and a test particle $M_t << M_1, M_2$. In the center of mass of the system (see Figure for geometry) the angular frequency is

$$\Omega = \sqrt{\frac{G(M_1 + M_2)}{D}} \ , \tag{1}$$

   where $D = x_{M1} + x_{M2}$. In this case we saw on week 2 that the derivative of Φ_{eff} is zero (i.e., the effective potential at a distance R form M_2 is zero). Remember that we can write the effective potential for a two dimentintal rotating potential as

$$\Phi_{eff} = \Phi - \frac{1}{2}|\vec{\Omega} \times \vec{x}| \ . \tag{2}$$

   Use that to derive the more exact expression for the Hill radius (i.e., with the $\sqrt{3}$) factor, under the assumption that $D >> R$.



   (b) (3pt) What were the two additional assumptions we used in the Hill derivation above?

3. In a homework assignment students were requested to consider a particle in a planetary ring with semi-major axis a that is in a circular orbit about a planet of mass M. In this system there is a nearby moon, also in a circular orbit around the planet with a semi-major axis a. The ratio of the moon to planet mass is $\mu = m/M << 1$. The difference between moon and particle semi-major axes (for the orbit around the planet) is $da << a$. At closes approach the moon gives a kick to the particle's velocity. The students were asked to calculate the kick velocity and worked together on the answer. A hint in the problem was that $m/M << a/da$.

(a) Student A got that

$$\delta v = \mu \left(\frac{a}{da}\right)^2 \sqrt{\frac{GM}{a}} \ , \tag{3}$$

What was the approximation she used? reproduce her derivation.

(b) Student B said that she is wrong to use this approximation. Who is correct?

4. **Derivation of the disruption relation** (24pt, 12pt each)

(a) Derive the following equations for the $\hat{\mathbf{r}}$ and $\hat{\phi}$ components from the close equations we wrote in class (i.e., momentum, continuity and poisson).

$$\hat{\mathbf{r}}: \quad \frac{\partial v_r}{\partial t} - 2\Omega\delta v_\phi = \frac{1}{\Sigma_0}\frac{dP_1}{dr} - \frac{\partial \Phi}{\partial r} \ . \tag{4}$$

$$\hat{\phi}: \quad \frac{\partial \delta v_\phi}{\partial t} + v_r\left(\Omega + \frac{d(\Omega r)}{dr}\right) = 0 \ . \tag{5}$$

(b) Using these equations find the dispersion relation we wrote in class.